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A New Blind Deconvolution Method of Single-Output the EEG Convolution Mixed Signals

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EEG signal has the chaotic character. After studying the statistic characteristics of chaotic signals, a deconvolution filter for chaotic signal is introduced by the linear predication error analysis. Based on the chaotic geometric characteristic, the output data of this filter are used to reconstruct the dynamical equation of the original chaotic signal. Then the output data are corrected according to the chaotic physical feature. So a blind deconvolution method is achieved successfully which has single-input and single-output chaotic convolution mixed signal. And EEG signal simulation result verified the effectiveness of this proposed method.

Key words: Blind deconvolution, chaos, linear predication, phase space reconstitution, EEG.

Chaos is a ubiquitous natural phenomenon in the complex non-linear systems, such as heart system, invertebrate nervous system ÿneuron network, ordinary differential equations, and planar non-linear mapping. So the extraction and the separation of the chaotic signals under the various the conditions are the very important topics in the signal processing. If the mechanism of the dynamics of the chaos system has been known, the method separating the chaotic signal from the mixed signals is to find a time sequence not only to meet the known rule of the chaos dynamics but also closes to the obtained observation sequence . But if the mechanism of the dynamics is unknown, the common method is phase space reconstitution (Goldberger *et al*., 1990; Liu, *et al*, .2002;Bell *et al*.,1995).

In practice, after transmitting, there will take place delay and attenuation on the chaotic signal is aroused by the channel. For example, the oceanic sound wave data include the reflectance information of the coast. The heart signal of the fetus has reflection and attenuation when it is transferred from the mother. The phenomena called the convolution. Thus under the common instances, the signal measured by the sensor is the linear combination of the source signal and the delayed and attenuated signal, that is the convolution mixed signal. Comparing with the instantaneous mixture, the solution to the convolution mixture is more difficulty, and is nearer the reality(Liu, *et al.,* 2002).

Obviously, knowing the chaotic dynamics mechanism ahead, the request can not meet for the blind separation. And after convolution with the system function, the chaotic signal is not homeomorphous with the original chaotic dynamic equation. So the original chaotic signals can no be directly recovered by adopting the phase space re-constitution.

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In this paper, the characteristics of the chaotic sequence are introduced firstly, and by using the similar random characteristic of the chaos and the bind signal processing technique, the deconvolution filter on chaotic signal is designed based on the prediction error analysis. And the output data of this filter are used to reconstruct the dynamical equation of the original chaotic signal. Then the output data are corrected according to the chaotic dynamical equation. So the blind deconvolution of the source signals and the transmission function in the single-input and single-output chaotic convolution system can be realized.

This method takes advantage of the chaotic physical features and the compensatory technique that is based on the phase space reconstruction chaos dynamic equation. And some EEG simulation results verified the effectiveness and adaptability of this proposed method.

Characteristics of the chaotic sequence

Definition of Chaos

If the non-linear system can be described by the differential equation whose variable is the time, the discrete form can be written as

$$x_{n+1} = f(x_n) \quad \text{... (1)}$$

A common definition of chaos is given by Devaney. Assuming (x,p) is a compact metric space, $f : x \rightarrow x$ is a continuous map, then we call that f is chaotic on X when f meets the following requirements: (1) f has the dependency on the initial value's sensitivity; (2) f is topologically transferred on X; (3) the period points of f are compact in X .

For example, Li-Yorke pointed out the non-linear differential equations based on the Logistic mapping.

$$x_{n+1} = \mu x_n(1 - x_n) \quad x_n \in [0.1] \quad \lambda \in [0,4] \quad \text{...(2)}$$

When μ=3.569945672, time period fork will appear, which will lead to the chaos.

Statistic Characteristics of the Chaotic Signals

Probability Distribution Function

Shuster H.G proved that the probability distribution function $\rho(x)$ in formula (2) is

$$\rho(x) = \begin{cases} \dfrac{1}{\pi\sqrt{x\sqrt{1-x}}} & 0 < x < 1 \\ 0 & else \end{cases} \quad \text{... (3)}$$

Here, $\rho(x)$ is non-related to the initial value $x(0)$, so Logistic map has periodicity.

The mean value $\bar{x}$ of the chaotic sequence.

$$\bar{x} = \lim_{N\to\infty} \frac{1}{N} \sum_{i=0}^{N-1} x(i) = \int_0^1 x\rho(x)dx = 0.5 \quad x \in (0,1) \quad \text{..(4)}$$

Auto-correlation Function

$$\begin{aligned} A(r) &= \lim_{N\to\infty} \frac{1}{N} \sum_{i=0}^{N-1} (x(i) - \bar{x})(x(i) + r - \bar{x}) \\ &= \int_0^1 x f^m(x)\rho(x)dx - (\bar{x})^2 \\ &= \begin{cases} 0.125 & r = 0 \\ 0 & r \neq 0 \end{cases} \\ m \quad &= 0.125\delta(r) \end{aligned} \quad \text{...(5)}$$

Where $f^m(x) = f(....f(x).....)$, and there are orders totally.

Cross-correlation Function

Selecting two sequences $x_1(n)$ and $x_2(n)$ arbitrarily in chaotic signal, then the cross-correlation function is

$$\begin{aligned} A(r) &= \lim \frac{1}{N} \sum_{i=0}^{N-1} (x_{1i} - \bar{x})(x_{2(i+r)} - \bar{x}) \\ &= \int_0^1\int_0^1 x_1 f^m(x_2)\rho(x_1)\rho(x_2)dx_1dx_2 - \bar{x}^2 \\ &= 0 \end{aligned} \quad \text{...(6)}$$

Power Spectrum

$$W(w) = \sum_{k=-\infty}^{\infty} A(r)e^{-rwk} = 0.125 \quad \text{...(7)}$$

Measure Entropy

Based on the measure entropy proposed by Kolmogorov (Haue, at el., 1994; Benettin, at el., 1976), we have

$$K_n = -\sum_{i_1, i_2 \cdots, i_n} \rho(i_0, i_1, \ldots i_n) \log_2 \rho(i_0, i_1, \cdots i_n) = \sum \lambda_i \quad \leq \lambda_1 \quad \text{.... (8)}$$

Where λ_1 is the maximal Lyapunov exponent. Under the Logistic attractor, when there are 10000 samples and the dimension of the embedding space is 5 , $\lambda_1 = (5.74 \pm 0.02) \times 10^{-1}$.

Comparing with the statistical characteristic of the white noise, we have the following conclusions:

(1) The statistical characteristic of the chaotic signals is similar with that of the white noise, so the chaotic signal has the similar random characteristic.

(2) Chaos is a special movement between the regular movement and the random movement. When the measure entropy $K = 0$, it is the regular movement, while $K = \infty$, it is the random movement, and when K is a positive constant, it is the decided chaotic movement. What is more, with the increase of K, it tends to the real random movement.

(3) To a chaotic system, the output is not convergent whose waveform is non-period and similar random.

Description of the method

A deconvolution filter on chaotic convolution mixed signal

Let $s(n)$ be the chaotic signal, $h(n)$ be the impulse response of the channel, and then the convolution model can be expressed as

$$y(n) = \sum_{n=0}^{\infty} h(n) s(t-n) \quad \text{... (9)}$$

Assume that the length of the observed chaotic convolution mixed signal $y(n)$ is , then design a p-order deconvolution filter and let $\hat{y}(n)$ be the estimation of $s(n)$.

Based on the linear prediction principle, the white noise sequence can be modeled by a linear difference equation. Because of the similar random character of the chaos, $\hat{y}(n)$ and $y(n)$ has the following relationship:

$$\hat{y}(n) = -\sum_{k=1}^{p} a_{pk} y(n-k) \quad \text{...(11)}$$

$$\hat{y}(n-p) = -\sum_{k=1}^{p} a_{pk} y(n-p+k) \quad \text{...(12)}$$

Because $\hat{y}(n)$ is the weighted linear combination of the data before $y(n)$, it is a forward prediction model. While $\hat{y}(n-p)$ is the weighted linear combination of the data after $y(n)$, it is a backward prediction model. Here, the forward error is

$$e_p(k) = y(n) - \hat{y}(n) = y(n) + \sum_{k=1}^{p} a_{pk} y(n-k) \quad \text{...(13)}$$

Based on the solution of the Yule-walker Equation proposed by Levinson (Benettin, *et al*, 1976), we have

$$a_{pk} = a_{p-1,k} + k_p a_{p-1,p-k} \quad \text{...(14)}$$

Then

1 $$e_p(n) = e_{p-1}(n) - k_p B_{p-1}(n-1) \quad \text{...(15)}$$

Where

$$e_{p-1}(n) = y(n) + \sum_{k=1}^{p-1} a_{p-1,k} y(n-k) \quad \text{...(16)}$$

$$B_{p-1}(n-1) = y(n-p) + \sum_{k=1}^{p-1} a_{p-1,k} y(n-p+k) \quad \text{...(17)}$$

In a similar way, the backward error can also be obtained.

$$B_p = y(n-p) - \hat{y}(n-p) = y(n-p) + \sum_{k=1}^{p} a_{pk} y(n-p+k) \quad \text{...(18)}$$

$$B_p = B_{p-1}(n-1) - k_p e_{p-1}(n) \quad \text{...(19)}$$

Combing formulas (15) and (19), we can find: when $p = 0$,

$$e_0(n) = B_0 = y(n) \quad \text{...(20)}$$

To minimize the forward error and the backward error simultaneously, we have

$$\partial E[e_p^2(n) + B_p^2(n)] / \partial k_p = 0$$

$H(w)$

$$\therefore \quad k_p = \frac{2\mathrm{E}\left[B_{p-1}(n-1)e_{p-1}(n)\right]}{\mathrm{E}[e_{p-1}^2(n)+B_{p-1}^2(n-1)]} \quad ...(21)$$

Based on the statistical character of the chaos, the chaotic signals have the ergodic character. So the statistical average can be substituted by the time average.

$$k_p = \frac{2\sum_n\left[B_{p-1}(n-1)e_{p-1}(n)\right]}{\sum_n[e_{p-1}^2(n)+B_{p-1}^2(n-1)]} \quad ...(22)$$

Then after iterative computations, the coefficients at different orders of the deconvolution filter aiming to the chaos can be obtained.

Setting of the Order of Chaotic Filter

The setting of order P of the chaotic filter is very important. Because the chaotic signal has the noise character according to the related literatures (Wang, at el., 2001; Cheng, at el., 2004), we can use the minimal descriptive length criterion.

$$\min(f(P)) = N\ln\sigma_p^{\,2} + \frac{P}{2}(\ln N) \quad ...(23)$$

Where σ_p is the prediction error, and with the increase of the data length N, it should tend to zero. When $f(P)$ puts up the obvious minimal value, P can be determined. But when the data length is short, the estimation of P is usually too high, and sometimes $f(P)$ will put up many minimal values. According to the characters of chaos, in a simpler form for (23), we choose P experientially

$$P \geq D \pm 1 \quad ...(24)$$

Where D is the number of the variables in the chaos dynamics system. In a word, the selection of P should experience many experiments until the results are satisfying.

Phase Space Reconstruction way of Chaotic Signal

Because chaotic signal is similar with noise but not the real noise, the recovered signal $\hat{y}(n)$ after chaotic filter is not as the same as the source signal $s(n)$, but it has the same form with $s(n)$. Based on chaotic dynamic theory, $\hat{y}(n)$ and $s(n)$ can be regarded as diffeomorphism. So by using $\hat{y}(n)$, the attractor of the chaotic system can be reconstituted. And then based on the geometric characteristics of the slick manifold of the chaotic attractorÿ the dynamic equation of this chaotic system will also be constituted. Finally the aim of recovering $s(n)$ can be obtained by contrast with data of the dynamic equation of this chaotic system.

According to the embedding theory proposed by Takens (Benettin, *et al.*, 2004), $s(n)$ and $\hat{y}(n)$ both have the same dimension. The slick manifold can be approximated by the partial tangential spaces of the points. The singular value decomposition (SVD) method introduced in literatures (Cheng, *et al.*, 2004) and (Cheng, *et al.*, 2012) is used in this paper. Assume d is the dimension of the reconstruction, $\hat{y}(n_i)\ i=1,2,\cdots,Q$ ($Q>d$) are the neighboring points in the neighboring domain Ω_n of $\hat{y}(n)$ when $n=n_0$, the estimation of $\bar{s}_0$ and the partial tangential space of $s(n)$ can be written respectively as the following:

$$\bar{s}_0 = \frac{1}{Q}\sum_{i=1}^{Q}\hat{y}(n_i) \quad ...(25)$$

$\bar{s}_0$

$$A=[\frac{\hat{y}(n_1)-\bar{s}_0}{\|\hat{y}(n_1)-\bar{s}_0\|},\ \frac{\hat{y}(n_2)-\bar{s}_0}{\|\hat{y}(n_2)-\bar{s}_0\|},\cdots\frac{\hat{y}(n_Q)-\bar{s}_0}{\|\hat{y}(n_Q)-\bar{s}_0\|}]$$
$$=[u_1,\ u_2,\ \cdots,\ u_d]\cdot\begin{bmatrix}\sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_d^2\end{bmatrix} 0_{d\times(Q-d)} \cdot\begin{bmatrix}v_1^T\\ v_2^T\\ \vdots\\ v_Q^T\end{bmatrix} \quad ...(26)$$

Where $\bar{s}_0$ is the center of Ω_n, and generally $\bar{s}_0 \neq \hat{y}(n_0)$. From the SVD of the matrix $\mathbf{A}$, the dimension l of the partial tangential space can be determined. When $\sigma_l^2 \gg \sigma_{l+1}^2$, we have $\sum_{i=1}^{l}\sigma_i^2 \approx \sum_{i=1}^{d}\sigma_i^2$. The origin of coordinate of the partial tangential space $T_{s(n)}M$ in the space R^d is , and one set of the orthogonal basis is $[u_1,\ u_2,\ \cdots,\ u_l]$. Then the point $s(n)$ in the manifold M corresponding to the reconstructed vector $\{\hat{y}(n)\}$ can be got

$$s'(n) = \bar{s}_0 + \mathrm{Proj}_{T_{\bar{s}_0}(M)}(\hat{y}(n)-\bar{s}_0) \quad ...(27)$$

Where $\mathrm{Proj}_{T_{\bar{s}_0}(M)}$ is the projection of to pass the point s_0 on M.

Let $U_l = [u_1, u_2, \cdots u_l]$, $\{u_i\}_{i=1}^{l}$ be the unit orthogonal basis of $T_{s(n)}M$, and $\tilde{U}_l = [\tilde{u}_1, \tilde{u}_2, \cdots \tilde{u}_l]$, $\{\tilde{u}_i\}_{i=1}^{l}$ be the unit orthogonal basis of $T_{s(n+1)}M$. The linear mapping: $T_{s(n)}M$ $T_{s(n+1)}M$ can be determined by LS algorithm. And the reconstruction dynamic system equation can be got (Cheng, *et al*., 2004):

$$s'(n+1) = U_l L U_l^T \hat{y}(n) + \hat{y}(n+1) - U_l L U_l^T s'(n) \quad ...(28)$$

The Blind Deconvolution Algorithm for single-input and single-output Chaotic Convolution Mixed Signal Based on Predication and Reconstruction Analysis

Discussion on the Chaotic Deconvolution

When research the chaotic deconvolution, we should pay attention to the following characters:

(1) Chaotic signal has the similar-Gauss character, which is of great benefit to the adoption of MLE algorithm, MMI algorithm, expanded H-J algorithm and the ARMA model [2, 10]. Because of the morbidity of the deconvolution, little error of the non-Gaussian kernel function can reduce the great changes of the solution.

(2) After convolution, the chaotic signal usually does not present the chaotic character any more. So the original chaotic signal can no be recovered directly by such method as the phase space reconstruction.

(3) Chaotic signal is not the real white noise, so the signal filtered by the chaotic deconvolution filter is not the optimal recover of the original chaotic signal. But the geometric characteristic of the slick manifold of the chaotic signal is important prior knowledge, and has the strong restrict on the solution. So the signal after blind deconvolution can be regularized according to this character of chaos

Description of the Algorithm

According to the physical features of chaos, the blind deconvolution method for single-input and single-output chaotic convolution mixed signal based on predication and reconstruction analysis is got. Followings are the steps of this method.

(1) According to the observation data $y(n)$, the estimated data $\hat{y}^0(n)$ can be given by using the chaotic deconvolution filter which be got by formulas (22) and (24).

(2) Reconstruct the dynamic system equation $s'(n)$ by using $\hat{y}^0(n)$ and formulas (28). Let $s'(n)$ and $\hat{y}^0(n)$ form the Poincare's sectionÿand to perform closing regulation for the points of $\hat{y}^0(n)$ departure $s'(n)$. Finally the reconstructive sequence $\hat{y}^1(n)$ can be obtained.

(3) Perform deconvolution by using $\hat{y}^1(n)$ and $y(n)$ to solve $\hat{h}^0(n)$

(4) To $\min\|y(n) - h(n) * \hat{y}^1(n)\|^2$, reconstruct $\hat{h}^0(n)$ into $\hat{h}^1(n)$.

(5) Repeat the steps (2), (3) and (4). When the resemble coefficients satisfy requirement, the iteration stops.

Above algorithm includes two deconvolutions and the further reconstruction and recover of the separating results. Because of the similar randomness of the chaotic signals, better signal $\hat{y}^0(n)$ after deconvolution filter can be got.

There are many methods to solve $\hat{h}^0(n)$. That is, we can use the deconvolution filter again or select other deconvolution methods. In this paper, the division algorithm in z-domain is adopted.

$$\hat{Y}(z) = H(z)Y(z)$$
$$\therefore \hat{H}(z) = \hat{Y}(z) / Y(z) \quad ...(29)$$

Perform FFT on the sequence directly to realize the z-transform, then adopt the division in the frequency domain, and to wipe influenced by the complex number, finally the real can be obtained.

To realize, $\min\|y(n) - h(n) * \hat{y}(n)\|^2$ and at the same time the diploid differences of the phase and amplitude of $\hat{h}^0(n)$ are not considered, so a weight function $d(n)$ can be used to amend, $\hat{h}^0(n)$ which can be written as

$$d(n) = \frac{1}{Q(n)} e^{j\Delta\Psi(n)} \quad ...(30)$$

Here, $\Delta\Psi(n)$ and $Q(n)$ are assumed values respectively. And the iterative equation is

$$h_{(n)}^{(k+1)} = d(n) h_{(n)}^{(k)} \quad ...(31)$$

When

$$\min \left\| y(n) - h^{(k+1)}(n) * \hat{y}(n) \right\|^2 < \varepsilon \quad ...(32)$$

The iteration stops, where ε is a threshold.

Based on the experimental experience, generally is a fixed constant, and $\Delta\Psi(n)$ is a positive phase offset.

Simulations

Experiment 1: Blind deconvolution of Chen s chaotic convolution signals

According to the Chen's equation

$$\begin{aligned} x' &= a(y - x) \\ y' &= (c - a)x - xz + cy \\ z' &= xy - bz \end{aligned} \quad (33)$$

When $a = 35, b = 3$ and $c = 28$ and the initial values $x(0) = -10, y(0) = 0$ and $z(0) = 27$ the chaos will be produced. And the one-dimension output can be regarded as the chaotic source signal $s(n)$. Let $h(n) = 0.1\sin\omega t, y(n) = s(n) * h(n)$ After the separation by the chaotic deconvolution filter (P=3) and a time the reconstruct regulation, the estimation $\hat{y}(n)$ of the source signal can be got. The waveforms of these signals are shown in Figure 1.

In Fig. 1, is the Poincare's section of $\hat{y}^0(n)$ and $s'(n)$. Using our method $\hat{h}(n)$ can also be got synchronously. There are similar coefficients of the output signals [7]: β(s(n), $y(n)$ =0.9929 β (h(n), $\hat{h}(n)$ = 0.9118.It shows that there is only the amplitude difference between and, but there is no phase difference.

While there are not only amplitude but also phase differences between and.

Experiment 2

Blind deconvolution of the EEG convolution mixed signals

EEG signal has the chaotic character [1]. Separating EEG signal from the convolution mixed signals has practical significance. Sample 100 points from the EEG signal as the sequence of the original signaland sample 30 negative points from Rayleign sequence as the system function $h(n)$,

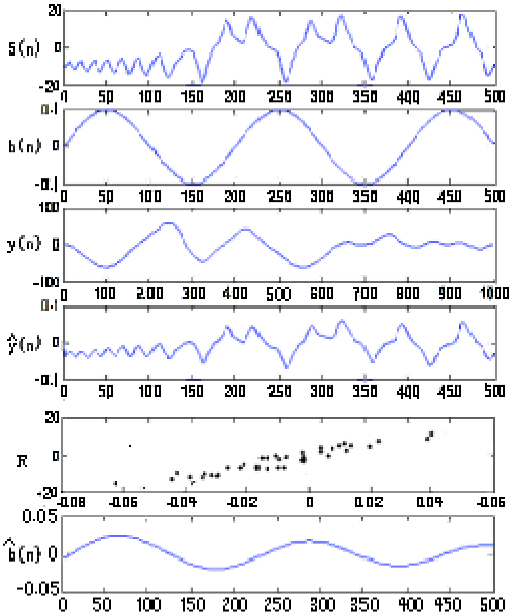

Fig. 1. Results of Experiment 1

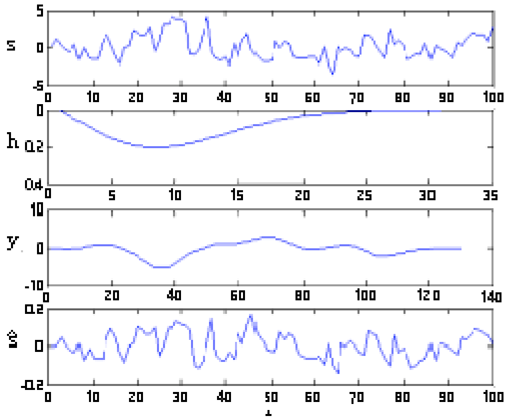

Fig. 2. Blind deconvolution Results of EEG

$y(n) = s(n) * h(n)$. Passing the deconvolution filter and the reconstruct regulation, the estimation $\hat{y}(n)$ of $s(n)$ can be got.

CONCLUSION

These results indicate that:

(1) The resemble coefficient is near 1, which means that the EEG separating result is good, and this can also be got from the resemble phase diagram.

(2) The length of influents the separating result greatly. The length of is nearer that of, the separating result is better.

(3) The order of the chaotic convolution filter is higher, the detail waveform in the separating results are more. That is, over compensation appears. So the selection of the order has great influence to the separating result.

(4) To increase of the time of the reconstruct has not effect clearly to improve similar coefficient.

Because chaotic signals as EEG are ubiquitous in the nature, the blind processing technique of the chaotic convolution mixed signals is practical. The prediction and reconstruction blind deconvolution algorithm proposed in this paper fully uses the physical features of the chaos. So this blind deconvolution algorithm aiming to special object has the practical meaning to expand the application of the blind signal separation and ulterior research on the character of these special signals.

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