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A Survey on Current Theory and Application of General Linear Image Processing

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General linear image processing (GLIP) framework, which is based on abstract linear mathematics, opens a new way to the development of image processing techniques by providing specific operations and structures for image representation and processing. This paper presents different models proposed so far for general linear image processing, including the logarithmic image processing (LIP), the general adaptive neighborhood image processing (GANIP), the logarithmic adaptive neighborhood image processing (LANIP)ÿthe parameterized logarithmic image processing (PLIP), the homomorphic logarithmic image processing (HLIP) and the pseudo-logarithmic image processing (Pseudo-LIP). After a description of each approach, we focus on their distinctive theory issues. Several LIP-model based application examples are exposed and discussed in image filtering, edge detection and morphological operation to show the practical advantage of LIP model. This study is helpful for an appropriate use of existing general linear image processing approaches and for systematically designing new approach.

Key words: Image processing, Image representation, General linear image processing, Logarithmic image processing.

In [1]developing the digital signal processing techniques, the linear signal processing system has been extensively studied and widely used, because it is mathematically easy to describe and analyze. Nevertheless, the linear signals processing systems is not necessary the best and even the right choice. Marr(Marr,1987) has pointed out that to develop an effective computer vision technique, three points must be considered, (1) why the particular operations are used, (2) how the signal can be represented, and (3) what implementation structure can be used. Myers(Myers, 1992) has recognized that there is no reason to persist with the classical linear operations, if via abstract analysis, more easily implemented and more generalized or abstract versions of mathematical operations can be created for image and signal processing.

Although the classic linear image processing (CLIP) framework still plays a central role in image processing, its usual definition of linearity and the conventional addition "ÿ" and scalar multiplication "×" operations suffer from drawback with respect to the non-linear images and digital images. For the non-linear images, the CLIP framework is not adaptive because the superposition of such images does not obey to the classical additive law.

Using the theory of abstract linear algebra make it possible to propose and examine entirely new operations and algebraic structures. Oppenheim and Stockham (Oppenheim *et al*.,1949; Oppenheim *et al*.,1967; Oppenheim *et al*., 1949;

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Stockham, 1972) introduced the multiplicative homomorphic image processing (MHIP) at the end of 1960s.And the logarithmic image processing (LIP) was developed by Jourlin and Pinoli(Jourlin, 1987; Jourlin, 1988) in mid-1980s. By introducing the general linear image processing framework into the general adaptive neighborhood (GAN) approach, Debayle and Pinoli(Panetta *et al*.,2007) presented both the general adaptive neighborhood image processing (GANIP) and the logarithmic adaptive neighborhood image processing (LANIP) to extend the originate model. In the context of specific applications, the parameterized LIP (PLIP), the homomorphic LIP (HLIP) and the pseudo-logarithmic image processing (Pseudo-LIP) were presented by Panetta and Wharton, Patrascu and Buzuloiu(Patrascu *et al*.,2003) and Vertan and Oprea(Vertan *et al*., 2008)respectively.

The article aims to present a study of current theory and application on general linear image processing (GLIP). In Section 2, the logarithmic image processing (LIP) model is introduced, focusing on its algebraic structures and mathematical notions. In Section 3, the applications of LIP model are illustrated. Next, Section 4 discusses the improved approaches, including the GANIP approach, the LANIP approach, the PLIP approach, the HLIP approach and the Pseudo-LIP approach. In Section 5, several LIP-model based application examples are presented in the area of image filtering, edge detection and morphological operation. Finally, in Section 6, conclusion and future research are proposed.

Logarithmic image processing model

This section demonstrates the symbols, operational rules, mathematical notions and structures of the logarithmic image processing (LIP) model. Aimed at summarizing the LIP model, the complete mathematical theory will not be detailed. First, the LIP gray tone function is introduced. Second, the vectorial structure on the gray tone function space is presented. Third, the gray tone space structures and the LIP fundamental isomorphism are illustrated. Finally, in order to define the LIP-Prewitt operator in Section 5, the differentiation of gray tone function is also described.

The Gray Tone Function Representation

In the LIP model, an intensity image *F* is represented by its associated gray tone function *f* (Pinoli, 1997). Such a function is defined on a non-empty spatial domain D in the Euclidean space R^2, called the spatial support, with values in the bounded real number interval [0, *M*), where *M* is strictly positive (in the digital case, *M* equal 256 for an 8-bit image). The value of a gray tone function at a spatial location (*x, y*) is called a gray tone, and the real number range interval [0, *M*) is thus called the gray tone range. The relationship between a gray tone function *f*(*x, y*) and its corresponding classical gray level function *F(x, y*) is given by:

$$f(x,y) = M - F(x,y) \qquad \text{...(2.1)}$$

Gray tone function has its physical meaning. In the context of transmitted light imaging processed, a gray tone function is considered to be a light intensity filter whose opacity is known at each point belonging to the spatial support *D*. The value 0 is reached at each point of *D* which is totally transparent, while the value *M* is reached at the point which is totally opacity.

The Vectorial Structure on the Gray Tone Function Space

The set of gray tone functions defined on the spatial support *D* and valued in the real number interval [0, *M*) is denoted *I*. The addition of two gray tone functions *f*(*x, y)* and *g(x, y)* and the scalar multiplication of *f*(*x, y*) by a positive real number are defined in terms of the usual real valued function operations as:

$$f(x,y) \oplus g(x,y) = f(x,y) + g(x,y) - \frac{f(x,y)g(x,y)}{M} \qquad \text{...(2.2)}$$

$$a \otimes f(x,y) = M - M\left(1 - \frac{f(x,y)}{M}\right)^{a} \qquad \text{...(2.3)}$$

These two specific operations (⊕ and) mathematically describe how images are combined and how the pixel value is amplified. The addition operation has been introduced (Jourlin, 1988)to be mathematically closed in the real interval [0, *M*) and additive, and to physically suit the case of transmitted light images. Actually, the addition expresses the transmittance product law, and the scalar multiplication has been built (Jourlin,1988)originated from the addition operation.

Proved by Pinoli, the set *I* is a positive cone for and , because it is closed for these specific algebraic operations. Either the addition of two gray tone function, or the scalar multiplication of a gray tone function by a positive real number results in a new gray tone function. This is a very desirable property for image processing.

In order to enlarge the positive linear cone I into a vector space, it is necessary to give a mathematical meaning to the opposite of a gray tone function $f(x, y)$, denoted by $\Theta f(x, y)$, and to extend the scalar multiplication to any real number . The opposite of a gray tone function $f(x, y)$ and the minus between two gray tone function $f(x, y)$ and $g(x, y)$ are defined as follows: According to this definition, the gray tone range is enlarged from $[0,M)$ to $(,M)$, and the positive restriction of the real number in the scalar multiplication operation is removed. What is more, the set of gray tone function, defined on the spatial support D and valued in the real interval $[,M)$, denoted G, with the operation and become a real vector space .

Figure1 shows the advantage of the LIP arithmetic over the conventional linear arithmetic. Figure 1(a) shows a landscape image, and a 4 4 block taken from this image. Figure 1(b) shows an arctic fox image and the same 4×4 block from this image. Figure 1(c) shows the image processed using linear addition, and the same 4×4 block from this image. Figure 1(d) shows the image processed using LIP addition, and the same 4×4 block taken from this image. In Figure 1(c), most of the pixels are outside the acceptable range for an image, and hence the resulting image is entirely white. On the other hand, the image in Figure 1(d) actually darkens instead of brightening. The result contributes to the manner in which the LIP addition processes the light information.

The Gray Tone Space, the Fundamental Isomorphism and Modulus Notion

With the specific operations and , the set of gray tones is a real vector space denoted E. It has been proved that, the gray tone function space is algebraically isomorphic to the classical vector space of function defined on the spatial support D and valued in the real number range (,bþ). The isomorphic mapping denoted is defined as follow:

$$\Theta f(x, y) = -M \frac{f(x, y)}{M - f(x, y)} \quad ...(2.4)$$

$$f(x, y) \Theta g(x, y) = M \frac{f(x, y) - g(x, y)}{M - g(x, y)} \quad ...(2.5)$$

According to this definition, the gray tone range is enlarged from $[0,M)$ to (H,M), and the positive restriction of the real number in the scalar multiplication operation is removed. What is more, the set of gray tone function, defined on the spatial support D and valued in the real interval $[H,M)$, denoted G, with the operation and become a real vector space .

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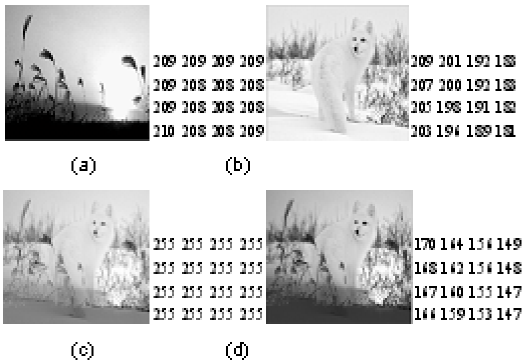

Fig. 1. (a) Landscape image (b) Arctic fox image (c) Linear addition of landscape and arctic fox (d) LIP addition of landscape and arctic fox

The Gray Tone Space, the Fundamental Isomorphism and Modulus Notion

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$$\varphi(f) = -M\ln(1-\frac{f}{M}) \quad ...(6)$$

According to Pinoli, the isomorphic mapping is called the fundamental isomorphism, and the isomorphic transform of a gray tone f is denoted by:

$$\bar{f} = \varphi(f) \quad ...(7)$$

Here, is a real number. The inverse isomorphic transformation is then defined as:

$$\varphi^{-1}(\bar{f}) = M(1-\exp(-\frac{\bar{f}}{M})) \quad ...(8)$$

The fundamental isomorphism has served as a powerful tool for developing the LIP model. With the fundamental isomorphism, notions and structures originated from functional analysis such as scalar product, Euclidean norm and Euclidean distance can be introduced:

Scalar product between gray tones

$$(f \mid g)_E = \varphi(f)\varphi(g) \quad ...(9)$$

Euclidean norm of gray tones

$$\|f\|_E = |\varphi(f)|_R \quad ...(10)$$

Euclidean distance between two gray tones

$$d_E(f,g) = |\varphi(f)-\varphi(g)|_R \quad ...(11)$$

Therefore, the gray tone space *E* is a Euclidean space, Banach space and metric space. Moreover, it has also been proven that the gray tone space is a Riesz space with the natural order relation, denoted*e”*, and the modulus $|\cdot|_E$, defined for a gray tone *f* by:

$$|f|_E = f, f \geq 0 \quad ...(13)$$

$$|f|_E = 0 \quad \ominus f, f \leq 0$$

The modulus notion plays a significant role in the LIP model, because for many gray tone functions its corresponding modulus is a positive gray tone and strong properties occur with respect to the vectorial operations

$$|f \oplus g|_E \leq |f|_E \oplus |g|_E \quad ...(14)$$

$$|a \otimes f|_E = |a|_R \otimes |f|_E \quad ...(15)$$

Here, *f* and *g* are two gray tones and *a* is a real number. The modulus notion appears as the accurate magnitude measure of gray tones(Pinoli, 1991) and has been used for the introduction of the contrast notion(Pinoli, 1991).

The Differentiation of Gray Tone Function

The LIP-model based differentiation has been introduced and studied [14] through the use of the general mathematical theory of differentiation of function valued in Banach space. In the sub-section, it was assumed that the interior set of the spatial support *D* is non-empty.

The direction derivative of a gray tone function *f*(*x, y*) at a point (*x, y*) along the direction of a non-zero plane vector $v=(v_x, v_y)$, denoted by $\partial_{\Delta v} f(x,y)$, is defined as follows:

$$\partial_{\Delta v} f(x,y) = \lim_{\substack{t>0 \\ t\to 0}} \frac{1}{t\|v\|_{R^2}} \otimes (f(x+tv_x, y+tv_y) \ominus f(x,y)) \quad ...(16)$$

Here, $||v||_R^2$ is the Euclidean norm of vector *v*. The definition is related to the usual directional derivative of the real valued function $\bar{f}(x,y)$, denoted by $\partial_v \bar{f}(x,y)$. The relation can be expressed as follow:

$$\partial_{\Delta v} f(x,y) = \varphi^{-1}(\partial_v \bar{f}(x,y)) \quad ...(17)$$

In the same way, the two first partial derivatives of a partially differentiable gray tone function *f*(*x*, y), denoted as $\partial_{\Delta x} f(x,y)$ and $\partial_{\Delta y} f(x,y)$ respectively.

The LIP model is a complete mathematical framework adapted to the study of gray tone function. Besides the algebraic structures and mathematical notions mentioned above, other algebraic and functional structures have been developed allowing powerful concepts and notions operating on special classes of gray tone function to be defined, such as integration, inner multiplication, metrics, norms, scalar product, correlation, convolution, Fourier and wavelet transformations and so on.

Applications

The LIP model have been applied to many fields of image processing include edge detection(Jourlin et, 1989; Wharton *et al*., 2008;), image enhancement(Jourlin *et al*., 1995; Deng, 2009), image segmentation(Palomares *et al*., 2006), image interpolation(Gremillet *et al*., 1994; Gremillet *et al*., 1995), image 3D reconstruction(Gremillet *et al*., 1995), contrast estimation(Brailean *et al*., 1991), image restoration(Deng and Cahill., 1993), image filtering(Deng,1993; Deng and Cahill., 1993), image multiscale decomposition (Deng and Cahill., 1993) and image data compression (Debayle *et al*., 2006; Patrascu *et al*., 2003), and so on.

The improved approaches

This section aims at developing an overview of the improved approaches, such as the general adaptive neighborhood image processing (GANIP) approach, the parameterized logarithmic image processing (PLIP) approach, the homomorphic logarithmic image processing (HLIP) as well as the pseudo-logarithmic image processing (Pseudo-LIP). For each approach, we mainly focus on their relevant mathematical operations and structures as well as the application issues, rather than an in-depth review.

The general adaptive neighborhood image processing (GANIP) approach

Debayle and Pinloi(Oppenheim *et al*., 1949)proposed a new image processing approach which is called general adaptive neighborhood image processing (GANIP) by introducing the GLIP frameworks to the spatially-adaptive image processing. In the GANIP approach, the operators are no longer spatially invariant but vary over the whole image with adaptive windows, taking intrinsically into account the local image features. A set of general adaptive neighborhoods (GANs set) is identified for each point in the image. A GAN is a subset of the spatial domain constituted by connected points whose measurement values, in relationship to a selected criterion such as luminance, contrast, thickness and curvature, fit within a specified homogeneity tolerance. These GANs are used as adaptive windows for image transformations or quantitative image analysis.

Compared with the adaptive neighborhoods (ANs sets), GAN is called general for two reasons. Firstly, the addition of a radiometric, morphological, or geometrical criterion in the definition of the usual AN sets allows a more significant spatial analysis to be performed. Secondly, both image and criterion mappings are represented in General Linear Image Processing (GLIP) frameworks(Panetta *et al*., 2007) allowing to choose a relevant structure consistent with the application to be addressed.

The parameterized logarithmic image processing (PLIP) approach

Although the LIP framework has been successfully used for many images processing area, there are still several limitations. Panetta *et al*. proposed a improve LIP model, called parameterization of logarithmic image processing model (PLIP), aiming to solve this problem encountered by the classic LIP approach.

The relevant mathematical operations are defined as follows:

$$v_1 +' v_2 = v_1 + v_2 - \frac{v_1 v_2}{\gamma(M)} \quad ...(3.1)$$

$$v_1 -' v_2 = k(M)\frac{v_1 - v_2}{k(M) - v_2 + \varepsilon} \quad ...(3.2)$$

$\varphi_2 = k(M)$

$$v_1 \times' v_2 = \varphi^{-1}(\varphi(v_1)\cdot\varphi(v_2)) \quad ...(3.3)$$

Where v is the gray tone function of the presented image , the parameters $\gamma(M)$, $k(M)$ and $\lambda(M)$ are linear function of the type $\gamma(M) = AM + B$, with A and B are constant parameters. Also, • is a very small constant introduced for the case where to avoid dividing by 0. Moreover, β is an exponential coefficient to the function used for multiplication.

The homomorphic logarithmic image processing (HLIP) approach.

Patrascu *et al*. (Patrascu *et al*., 2007) proposed an improved logarithmic image processing technique, called homomorphic LIP model. This method transform the image's gray tone value bound in [0, *M*] into the standard set (-1,1), using the translation below:

$$v = \frac{2}{M}\left(u - \frac{M}{2}\right) \quad ...(3.4)$$

Where $u \in [0, M)$ and $v \in (-1, 1)$.

The $E=(-1,1)$ interval plays the central role in the proposed model: it is endowed with the structure of a linear space over the scalar field of real numbers R. In this space, the basic operations are directly defined as follows:

$$\forall v_1, v_2 \in E, v_1 +' v_2 = \frac{v_1 + v_2}{1 + v_1 v_2} \quad ...(3.5)$$

$$\forall v_1, v_2 \in E, v_2 -' v_1 = \frac{v_2 - v_1}{1 - v_1 v_2} \quad ...(3.6)$$

$$\forall v \in E, \lambda \in R, \lambda \times' v = \frac{(1+v)^\lambda - (1-v)^\lambda}{(1+v)^\lambda + (1-v)^\lambda} \quad ...(3.7)$$

The pseudo-logarithmic image processing (Pseudo-LIP) approach

Pseudo-Logarithmic Image Processing model was proposed by Vertan *et al*, allowing the computation of gray-level addition, subtraction and multiplication with scalars within a fixed gray-level range [0, *D*] without the use of clipping.

The basic operations of this model are defined as followed:

$$\forall v_1, v_2 \in [0,1], v_1 +' v_2 = \frac{v_1 + v_2 - 2v_1 v_2}{1 - v_1 v_2} \quad ...(3.8)$$

$$\forall v_1, v_2 \in [0,1], v_2 -' v_1 = \frac{v_2 - v_1}{1 + v_1 v_2 - 2v_1} \quad ...(3.9)$$

$$\forall \lambda \in R, v \in [0,1] \quad \lambda \times' v = \frac{\lambda v}{1 + (\lambda - 1)v} \quad ...(3.10)$$

Vertan *et al*. showed the implementation of classical edge detection techniques under the proposed approach yields significant superior performance as compared with the classical operations. Moreover, as proposed by Vertan and his colleagues, the pseudo-logarithmic image processing model based edge detection method can be extended to any derivative-type edge detection technique.

Practical examples

Experimental comparative results are given in this section to illustrate the advantages of the LIP approach over the traditional approach. The addressed image processing area includes image filtering, edge detection and general adaptive neighbourhood based morphological operation. The original test image is that of Lena which is a reflected light image. This image is dignified onto 8 bits and is defined on 256×256 pixels.

Application to Image Filtering

In the field of image processing, image filtering is a necessary preliminary step in image pre-processing, such as denoising, restoration, enhancement, pre-segmentation and sharpening. This sub-section aims to demonstrate the LIP-model based image filtering method has a better performance compared with the tradition method. We choose Mean Filter for this section. Mean Filter can be expressed as

$$a(i,j) = \frac{1}{n \times n} \sum_{k=i-n/2}^{i+n/2} \sum_{l=j-n/2}^{j+n/2} f(k,l) \quad ...(4.1)$$

Where $a(i,j)$ is the result of the processing image, n is the size of the processing window, $f(k,l)$ is the gray value of the pixel in the $n \times n$ window.

Figure 2 shows that the LIP-based mean filtering approach overcome the traditional mean filtering. Indeed, 'Lena' after mean filtering based on LIP can achieve better visual quality than that of the traditional mean filtering.

The table below shows the PSNR of the two methods discussed above according to differently types of noise.

Since a larger PSNR means a better performance, from Table1 we can learn that the LIP-model based mean filtering yields superior performance over the traditional mean filtering.

Fig. 2. Image filtering by applying the LIP-model based mean filtering and traditional mean filtering techniques on Lena's image. (a) original Lena's image; (b) The blurry image by gaussian noise with the variance is 0.005; (c) the result of traditional method; (d) the result of LIP-based method

Table 1. The PSNR of LIP-model based mean filtering (MF) and the traditional mean filtering according to different kinds of noise

	Gaussian noise	Poisson noise	Speckle noise
CLIP MF	28.430	27.963	28.203
LIP MF	28.587	28.057	28.341

Application to Morphological Operation

This sub-section aims to demonstrate the overwhelming advantage of the GANIP model. In order to evaluate the approach, a comparative study is proposed between the general adaptive and conventional morphological operators.

We define a mathematical morphology dilate operator based on GAN sets with the 'luminance' criterion. We set the value of the GAN set tolerance m=5. Figure 3 shows the experiment results. In order to compare these two methods, we add some gaussian noise to the original image. The traditional dilate structure element is *5×5* square structure. As showed above, traditional dilate damaged the spatial structure contrary to the GAN operators. The GAN set dilate perform better and get a superior visual image. But GAN set dilate need to compute adaptive neighborhood of each pixel of the image, it has higher time complexity.

Fig. 3. The experiment results of GAN set dilate and traditional dilate. (a) original image; (b) the image blurry by guassian noise; (c) the result of traditional dilate; (d) the result of GAN sets dilate

The traditional dilate structure element is 5×5 square structure. As showed above, traditional dilate damaged the spatial structure contrary to the GAN operators. The GAN set dilate perform better and get a superior visual image. But GAN set dilate need to compute adaptive neighborhood of each pixel of the image, it has higher time complexity.

CONCLUSION

The LIP approach was proposed to define an additive operation closed in the bounded real number range, which is mathematically well defined, physically consistent with concrete physical and practical image setting, and available for the introduction of an abstract ordered linear topological and functional framework. During the last decades, some improved approaches were proposed. By introducing the current GLIP framework into adaptive neighborhood image processing, both the GANIP and the LANIP was developed. Moreover, for specific purpose, some other improved approaches like the PLIP, the HLIP as well as the pseudo-logarithmic image processing were presented.

Although the proposed models based on GLIP frameworks have many appreciated advantages, they still suffer from several limitations. For example, regarding the LIP approach, when two visually pleasing images are added together the output image may not be visually pleasing. Therefore, the improved models based on the current GLIP framework will be a hot research field in the future. And, due to the fact that most of the presented models are focusing on the gray image, color image processing technique based on the GLIP framework still need to be studied. Additionally, when it comes to the recently proposed models like GANIP and LANIP, many potential applications are still existed for further research.

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REFERENCES

1. Brailean JC, Little D, Giger ML, Chen CT, Sullivan BJ., A quantitative performance evaluation of the EM algorithm applied to radiographic images. Proc. SPIE Biomedical Image Processing II 1991; **1450**: 40-46.
2. Debayle J and Pinoli JC., General adaptive neighborhood image processing: Part I: introduction and theoretical aspects. *Journal of Mathematical Imaging and Vision*. 2006; **25**(2): 245–266.
3. Deng G., Image and signal processing using the logarithmic image processing model. Ph.D. thesis, Dept. of Electronic Engineering, La Trobe University, Australia, 1993.
4. Deng G., An Entropy Interpretation of the Logarithmic Image Processing Model With Application to Contrast Enhancement. IEEE *Transactions on image processing*. 2009; **18**(5): 1135-1140.
5. Deng G, Cahill LW., Generating sketch image for very low bit rate image communication. in: Proc. 1st IEEE Australian and New Zealand Conf. on Intelligent Information Systems, Perth Australia, 1993; 407-411.
6. G. Deng, L.W. Cahill, "A novel multiscale image filtering algorithm using the contrast pyramid," in: Proc. 2nd Internat. Conf. on Simulation and Modelling, Melbourne, Australia, 1993; 57-65.
7. Gremillet P, Jourlin M, Pinoli JC., LIP-model-based three-dimensional reconstruction and visualization of HIV infected entire cells. *J. Microsc.* 1994; **174**: 31-38.
8. Gremillet P, Coudert JL, Jourlin M, Pinoli JC, Three dimensional image reconstruction and visualization of human jaws. in: Proc. IEEE Workshop on Nonlinear Images and Signals, *Halkidiki, Greece,* 1995; 2&22: 819-822.
9. Jourlin M,Pinoli J.C., Logarithmic image processing, Acta Stereol. 6:651-656.
10. Jourlin M,Pinoli J.C (1988), A model for logarithmic image processing, *Microsc J*. 1987; **149**(1):21-35.
11. Jourlin M, Pinoli, JC, Contrast definition and contour detection for logarithmic images. *Journal of Microscopy*. 1989; **156**: 33-40.
12. Jourlin M, Pinoli JC., Image dynamic range enhancement and stabilization in the context of the logarithmic image processing model. *Signal Processing*. 1995; **41**: 225-237.
13. Marr D., A Computational Investigation into the Human Representation and Processing of Visual Information, BA *Science*.1987; **236**:448-450.
14. Myers D.G, Digital Signal Processing Efficient Convolution and Fourier Transform Technique, *The Journal of the Acoustical Society of America.* 1992; **91**(1):536-536.
15. Oppenheim A.V, Schafer R.W,Stockham T.G, Jr., Nonlinear filtering of multiplied and convolved signals, Proc. Audio and Electroacoustics, *IEEE Transactions* 1949; **16**(3): 1264-1291.
16. Oppenheim A.V, Generalized superposition, *Inform. Control* .1967; **11**: 528-536.
17. Oppenheim A.V, R.W. Schafer,T.G. Stockham, Jr., Nonlinear filtering of multiplied and convolved signals, *Proc. IEEE*. 1949; **56**:1264-1291.
18. Palomares JM, Gonzalez J, Ros E, Prieto A., General Logarithmic Image Processing Convolution. *IEEE Transactions on Image Processing*. 2006; **15**: 3602-3609.
19. Panetta K, Wharton E, Agaian S., Parameterization of Logarithmic Image Processing Models. IEEE Tran Systems, Man, and Cybernetics, 2007.
20. Patrascu V, Buzuloiu V, Vertan C, Fuzzy image enhancement in the framework of logarithmic model, In: Nachtegael, M., Kerre, E. (eds.) Fuzzy Filters for Image Processing. *Studies in Fuzziness and Soft Computing*. 2003; **122**: 219–237.
21. Patrascu V, Buzuloiu V, Vertan C, Fuzzy image enhancement in the framework of logarithmic model. In: Nachtegael, M., Kerre, E. (eds.) Algorithms in Modern Mathematics and Computer Science. *Studies in Fuzziness and Soft Computing*. 2003; **122**: 219-237.
22. Pinoli J.C, The Logarithmic Image Processing Model: Connections with Human Brightness Perception and Contrast Estimators. *Journal of Mathematical Imaging and Vision*.1997; **7**: 341-358.
23. Pinoli J.C, A contrast definition for logarithmic images in the continuous setting, *Acta Stereol.* 1991; **10**: 85-96.
24. Stockham T.G, Jr., Image processing in the context of a visual model,"Proc. IEEE.1972; **60**: 828-842.
25. Vertan C, Oprea A, Florea C and Florea L, A Pseudo-logarithmic Image Processing Framework for Edge Detection Blanc-Talon *et al*. 2008; **5259**: 637–644.
26. Wharton EJ, Panetta K, Agaian SS., Logarithmic edge detection with applications. *Jour. of Computers.* 2008; **3**(9): 3346-3351.